Jarrow-Lando-Turnbull Version 1.0.0

1 Introduction

This plug-in implements the Jarrow-Lando-Turnbull (JLT) model which may be used to price bonds with default risk, i.e., bonds for which there is the possibility that the issuer does not honor the schedule of payments. From risky zero coupon bond prices, one can calculate credit spreads. JLT is a reduced-form model, i.e., it models the credit migration process (which is the driver of the credit risk) through a Markov Chain with the default state being absorbing. It takes as input for the credit migration process the credit transition matrices published by rating agencies. After proper calibration, the model provides a term structure (zero coupon bond prices) for each rating class considered.

2 Usage in Fairmat

This plug-in is used in conjunction with the Fairmat Economic Scenarios Generator, which automatically uses this model when installed in the system. For more details see the ESG documentation.

3 The JLT model

3.1 Description

The primary output of the JLT model is a formula to price zero-coupon bonds which have some degree of credit risk. The credit risk is modeled by means of the information contained in credit transition probability matrices. The JLT pricing formula for risky zero-coupon bonds is:

$$v^i(t,T) = p(t,T)[\delta + (1-\delta)\tilde{Q}_t(\tau > T)].$$

This pricing formula is flexible with regard to the model one wants to use for the nominal risk-free interest rates. Indeed, the riskless zero coupon bond prices, $p(t,T)$, can be obtained from any model the user is interested in (one factor Hull-White, Black-Karasinsky, LMM, two factor Hull-White, etc.) In Fairmat ESG the model for nominal risk-free interest rates is the Gaussian Squared Pelsser model, due to the fact that it guarantees no negative interest rates, a feature we deem important in this current low interest rate environment and because it is amenable to analytical option pricing formulas like one factor Hull-White.

$\tilde{Q}_t(\tau > T)$ is a Markov Chain transition matrix under the risk neutral measure from which survival probabilities (conditional on survival at time t) for each rating class and for every maturity T can be read out of the last column of $\tilde{Q}_t(\tau > T)$ matrices.

δ is an exogenously given constant which represents the part of the face value of the bond that will be recuperated in case of default, i.e it is a recovery rate. It can be estimated from historical data and it can be specified for each rating class. However, for the sake of simplicity and in line with the spirit of the original JLT paper (1997) we have assumed δ=40% for all rating classes.

The super index i in the $v^i(t,T)$ corresponds to each rating category included in the model. In our case, i will run from i=1, corresponding to Aaa to $i = 7$, corresponding to the Caa rating. $i = 8$ corresponds to the default state.

JLT admits both a discrete-time and a continuous-time modelling. We have chosen to develop the continuous time setting due to its greater accuracy.

From the $v^i(t,T)$ one can calculate instantaneous forward rates. Within a continuous-time JLT model, forward rates are given by this formula:

$$f^i(t,T) = \frac{-\partial}{\partial T} log v^i(t,T)$$

plugging into this formula the JLT expression for ZC bond prices, $v^i(t,T)$, and some algebra yields this expression:

$$f^i(t,T) = f(t,T) - \frac{\delta + (1-\delta)\frac{\partial}{\partial T}\tilde{Q}_t(\tau > T)}{\delta + (1 - \delta\tilde{Q}_t(\tau > T)}$$

from where the spreads are obtained as:

$$spd^i(t,T) = f^i(t,T) - f(t,T)$$

3.2 Required market data and preprocessing

The input data that is necessary to the JLT model is:

- `The one-year historical credit transition probability matrix.`
- `The risk-free zero-coupon bond prices curve.`
- `The risky zero-coupon bond prices curve for every credit rating category considered in the model.`
- `Recovery rate in the event of default of a bond.`

One-year historical credit transition probability matrices are provided by rating agencies such as Moody's or S&P. Matrices with not totally up-to-date figures can be found in the internet.

The risk-free zero-coupon bond prices curve can be obtained directly from Bloomberg or stripped out from the euro-swap rates quotations obtainable from Bloomberg as well.

The recovery rate in case of default is the percentage of the nominal of the bond that the bondholder will collect if the bond defaults. As we have previously said, in a first approximation it is taken as constant and equal for all rating classes.

The risky zero-coupon bond prices curves need to be stripped out from quoted market prices of coupon-bearing bonds. Since we are using a credit transition matrix with the rating classes: Aaa, Aa, A, Baa, Ba, B and Caa, we have worked out a zero-coupon curve for each one of those 7 rating categories and for maturities up to 20 years.